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MEMORANDUM REPORT BRL-MR-3894

BRL

MEASUREMENTS OF RANGE, DEFLECTION, AND HEIGHT OF BURST FOR FIRED ARTILLERY SHELL, METHOD II - A LEAST-SQUARES METHODOLOGY

NEAL P. ROBERTS

FEBRUARY 1991

U.S. ARMY LABORATORY COMMAND

BALLISTIC RESEARCH LABORATORY
ABERDEEN PROVING GROUND, MARYLAND

UNCLASSIFIED

REPORT DOCUMENTATION PAGE			Form Approved OMB No 0704-0188

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE February 1991	3. REPORT TYPE AND DATES COVERED Final, Jan 80 - Jan 90

4. TITLE AND SUBTITLE
Measurements of Range, Deflection, and Height of Burst for Fired Artillery Shell, Method II - A Least-Squares Methodology

5. FUNDING NUMBERS
PR: 1L162618AH80

6. AUTHOR(S)
Neal P. Roberts

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
US Army Ballistic Research Laboratory
ATTN: SLCBR-DD-T
Aberdeen Proving Ground, MD 21005-5066

10. SPONSORING/MONITORING AGENCY REPORT NUMBER
BRL-MR-3894

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited

12b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)

An alternative mathematical method for obtaining measurements of range, deflection, and height of burst for a fired artillery shell is explained. The associated algorithms and computer program are illustrated, discussed, and listed. A comparison of this methodology with triangulation is also made.

14. SUBJECT TERMS
Measurements, Range, Deflection, Height of Burst, Time of Flight, Triangulation, Least-Squares Method

15. NUMBER OF PAGES 46

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18 SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UL

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ACKNOWLEDGMENTS

The author would like to acknowledge certain people who aided me in this project—namely, Mrs. Agnes Kodat, Mr. Jack, and Mr. Dennis Flaherty, all from the Combat Systems Testing Agency (CSTA), Aberdeen Proving Ground, MD; my reviewers, Mr. Robert McCoy and Dr. Gene Cooper; and my supervisor, Mr. Robert Lieske. Mrs. Kodat and Mr. Jack graciously provided me with the observational input data of Appendix C. Mr. Flaherty graciously provided me access and gave me some insights into his triangulation computer program. Mr. McCoy and Dr. Cooper provided me with worthwhile comments. And Mr. Lieske provided me with the least-squares algorithm and mathematical model. My thanks to all these people.

My efforts in writing this report are dedicated to my mother, my Uncle Tony (her brother), my Aunt Etta (her sister), and all the other good people of the state of Connecticut, U.S.A. They are my strength and inspiration.

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1. INTRODUCTION

The purpose of this report is to explain a new mathematical method of measuring the range, deflection, and height of burst for a fired artillery shell. This new mathematical procedure can be used as an alternative to triangulation (Roberts 1990). It can be employed when there are three or more tower readings of azimuth and elevation on a fired projectile.

In triangulation, when three or four tower readings of azimuth and elevation are observed and the smallest optimum triangle whose area is less than 50 square meters is found, range, deflection, and height of burst are computed as follows. Range is the distance from the gun to center of "impact point" in the found triangle. Deflection from line of fire is computed as a perpendicular distance from "impact" to the line of fire and as an angle between line of fire and vector from weapon to "impact point." Altitudes are computed from each tower using the associated elevation and tower height and are then averaged (Roberts 1990).

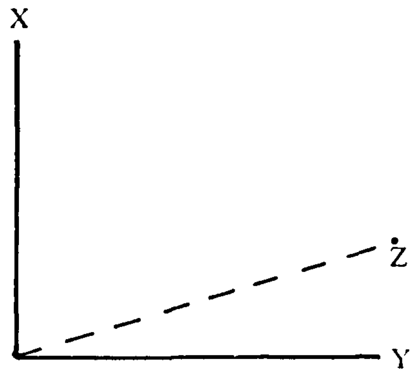
The new technique operates differently. A least-squares method which minimizes angular error is employed iteratively to find the range, deflection, and height of burst point while making use of the same data input to triangulation. The procedure is used for all tower readings as well as for all combinations when one tower reading is omitted. Then, statistically, error computations and associated burst points can be compared to see if there is significant difference. Thus, this method provides a means of detecting a bad tower azimuth/elevation observation in the process of using all the information one has access to.

This report will first present the algorithms used in this technique. Then, a discussion of the algorithms and associated computer programs (Appendices A, B, C, and D) will follow. The conclusion will discuss the results of computer runs by both triangulation and the new technique applied to the same observational data input.

2. ALGORITHMS

2.1 Least-Squares[†] Algorithm (developed by Mr. Robert Lieske and used in subroutine LSTQR in least-squares computer program).

[†] For more information on least-squares, see "Introduction to Statistical Theory," by P. G. Hoel, S. C. Port, and C. T. Stone, 1971.



(X_i, Y_i, Z_i) - position coordinates of burst point formed by two intersecting tower azimuth readings and an elevation from one tower.

Step 1. For burst point position (X_i, Y_i, Z_i), compute root mean square errors in azimuth (ERMS) and elevation (ERMSEL) from n tower reading locations.

Let,

$\Delta\ AZ_k$ = azimuth angle (of segment from tower k to burst point) - AZ(I,K) (azimuth reading of round from tower k), and

$\Delta\ EL_k$ = elevation angle (of segment from tower k to burst point) - ALZ(I,K) (elevation reading of round from tower k).

Then,

$$ERMS = \left[\sum_{k=1}^{N} \frac{(\Delta\, AZ_k)^2}{n}\right]^{1/2} \quad ; \quad ERMSEL = \left[\sum_{k=1}^{N} \frac{(\Delta\, EL_k)^2}{n}\right]^{1/2} \quad . \tag{1}$$

Step 2. Find ΔX, ΔY, ΔZ so that the following quantities are minimized where from a tower position (X(k),Y(k),Z(k)), one has for k = 1,..,n:

$$X_k = X_i - X(k);$$
$$Y_k = Y_i - Y(k);$$
$$Z_k = Z_i - Z(k);$$

$$E^2 = \sum_{K=1}^{N} \left[\Delta AZ_k - \frac{\partial AZ_k}{\partial \Delta X}\, \Delta X - \frac{\partial AZ_k}{\partial \Delta Y}\, \Delta Y - \frac{\partial AZ_k}{\partial \Delta Z}\, \Delta Z\right]^2 \quad . \tag{2}$$

$$E'^2 = \sum_{K=1}^{N} \left[\Delta EL_k - \frac{\partial EL_k}{\partial \Delta X} \Delta X - \frac{\partial EL_k}{\partial \Delta Y} \Delta Y - \frac{\partial EL_k}{\partial \Delta Z} \Delta Z \right]^2 \tag{3}$$

Take the partial derivative of E^2 in Equation 2 with respect to ΔX and ΔY and take the partial derivative of E'^2 in Equation 3 with respect to ΔZ. Set each partial derivative to zero and rearrange terms to give the following equations:

$$\sum \Delta AZ_k \left(\frac{\partial AZ_k}{\partial \Delta X}\right) = \sum \left(\frac{\partial AZ_k}{\partial \Delta X}\right)^2 \Delta X + \sum \left(\frac{\partial AZ_k}{\partial \Delta Y}\right)\left(\frac{\partial AZ_k}{\partial \Delta X}\right) \Delta Y + \sum \left(\frac{\partial AZ_k}{\partial \Delta Z}\right)\left(\frac{\partial AZ_k}{\partial \Delta X}\right) \Delta Z \; ; \tag{4}$$

$$\sum \Delta AZ_k \left(\frac{\partial AZ_k}{\partial \Delta Y}\right) = \sum \left(\frac{\partial AZ_k}{\partial \Delta X}\right)\left(\frac{\partial AZ_k}{\partial \Delta Y}\right) \Delta X + \sum \left(\frac{\partial AZ_k}{\partial \Delta Y}\right)^2 \Delta Y + \sum \left(\frac{\partial AZ_k}{\partial \Delta Z}\right)\left(\frac{\partial AZ_k}{\partial \Delta Y}\right) \Delta Z \; ; \tag{5}$$

$$\sum \Delta EL_k \left(\frac{\partial EL_k}{\partial \Delta Z}\right) = \sum \left(\frac{\partial EL_k}{\partial \Delta X}\right)\left(\frac{\partial EL_k}{\partial \Delta Z}\right) \Delta X + \sum \left(\frac{\partial EL_k}{\partial \Delta Y}\right)\left(\frac{\partial EL_k}{\partial \Delta Z}\right) \Delta Y + \sum \left(\frac{\partial EL_k}{\partial \Delta Z}\right)^2 \Delta Z \, . \tag{6}$$

Note the following terms can be derived:

$$\frac{\partial AZ_k}{\partial \Delta X} = - \frac{Y_k}{X_k^2 + Y_k^2} \quad , \quad \frac{\partial AZ_k}{\partial \Delta Y} = \frac{X_k}{X_k^2 + Y_k^2} \quad , \quad \frac{\partial AZ_k}{\partial \Delta Z} = 0 \qquad (k=1,..,n) \; ;$$

$$\frac{\partial EL_k}{\partial \Delta X} = \frac{-X_k \; Z_k}{(X_k^2+Y_k^2+Z_k^2)(X_k^2+Y_k^2)^{1/2}} \qquad (k=1,..,n) \; ;$$

$$\frac{\partial EL_k}{\partial \Delta Y} = \frac{-Y_k \quad Z_k}{(X_k^2+Y_k^2+Z_k^2)(X_k^2+Y_k^2)^{1/2}} \qquad (k=1,..,n) \; ;$$

$$\frac{\partial EL_k}{\partial \Delta Z} = \frac{-(X_k^2+Y_k^2)^{1/2}}{(X_k^2+Y_k^2+Z_k^2)} \qquad (k=1,..,n) \; .$$

With the above terms substituted into Equations 4, 5, 6, one notices that Equations 4 and 5 become two equations in the two unknowns ΔX, ΔY, and Equation 6 is one equation in the unknowns ΔX, ΔY, ΔZ.

Solve Equations 4 and 5 simultaneously to obtain ΔX, ΔY. Substitute the values ΔX, ΔY just obtained into Equation 6 so the ΔZ can be solved for. One has just computed ΔX, ΔY, ΔZ so that Equations 2 and 3 have been minimized.

Step 3. If ΔX, ΔY, ΔZ < .1, then the point taken as the burst point is (X_i, Y_i, Z_i). Otherwise, compute new X_i, Y_i, Z_i values as follows:

$$X_i = X_i - \Delta X;$$
$$Y_i = Y_i - \Delta Y;$$
$$Z_i = Z_i + \Delta Z;$$

and then repeat steps 1, 2, and 3. If the process does not converge in 200 iterations, a message, "DOES NOT CONVERGE," is printed out.

This algorithm can be applied to the case of all towers and to the case where each combination of one tower is omitted.

2.2 Modified F-Test Algorithm[†] With Outlier Scheme - (used in subroutines MODF and OUTLY in least-squares computer program).

[†] For more information on the modified F-Test algorithm, see "Introduction to Statistical Analysis," by W. J. Dixon and F. J. Massey, 1957.

Using algorithm 2.1), one obtains the following values:

ERROR (J) - Erms (azimuth)
ERRORE(J) - Erms (elevation - el)
XCNT (J) - X coordinate of burst point
YCNT (J) - Y coordinate of burst point
ZCNT (J) - Z coordinate of burst point

when omitting tower J (J = 1,..,n) and when all towers are used the following:

ERROR (JALL) - Erms (azimuth)
ERRORE(JALL) - Erms (elevation - el)
XCNT (JALL) - X coordinate of burst point
YCNT (JALL) - Y coordinate of burst point
ZCNT (JALL) - Z coordinate of burst point

Step 1. Find smallest ERROR(J) that is less than ERROR(JALL). If any, conduct modified F-test with outlier scheme as follows:

$$\bar{x} = \sum_{K=1}^{n+1} \frac{ERROR(K)}{n+1} ;$$

$$\overline{x^{J}} = \sum_{\substack{K=1 \\ K \neq J}}^{n+1} \frac{ERROR(K)}{n} ;$$

$$S = \sum_{K=1}^{n+1} (ERROR(K) - \bar{x})^2 ;$$

$$S' = \sum_{\substack{K=1 \\ K \neq J}}^{n+1} (ERROR(K) - \overline{x^{J}})^2 ;$$

$$F_{n+1} = \frac{S'}{S} .$$

Step 2. Check to see if F_{n+1} derived from modified F-test is significant (i.e., is an outlier) using n+1 degrees of freedom. Subroutine OUTLY in least-squares computer program has data statements giving criterion for 10%, 5%, 2.5% and 1% levels of significance and up to 8 degrees of freedom (Spiegel 1975; Hoel, Port, and Stone 1971).

If ERROR(J) is found to be significantly smaller then ERROR(JALL), then along with range, deflection, burst height (HOB), and error values of burst point found using all towers, similar values for burst position found with tower J omitted are also printed out.

Step 3. Find smallest ERRORE(J) that is less than ERRORE(JALL). If any, conduct modified F-test with outlier scheme in a manner similar to Steps 1 and 2.

3. DISCUSSION

After running the least-squares computer program (Appendix B) modified with special write statements on the sample observational data input of Appendix C, the process of convergence of the least-squares algorithm was investigated. It was found that when using all towers or when a tower was omitted, convergence took place in three or less iterations of the subroutine LSTQR. Also, the root mean square errors had the tendency to reduce in the iteration process. This, of course, is expected. When the convergence process was begun at a different initial burst point (X_i, Y_i, Z_i) by changing the combination of intersecting tower reading azimuths and tower elevation, it ended at the same point. That is, although the initial (X_i, Y_i, Z_i) of the convergent process was changed, the final (X_i, Y_i, Z_i) was invariant and produced the same identical output in Appendix D.

It must be admitted that the convergent process of the least-squares algorithm was not studied in great detail. A statement of theorem dealing with convergence or divergence of the algorithm was unable to be made and proven by the author. Perhaps some pure mathematician who reads this report will make such an attempt.

4. CONCLUSION

The least-squares computer program which utilizes the least-squares algorithm is contained in Appendix B with associated information in Appendix A. This computer program along with Dennis Flaherty's triangulation computer program has been applied to the same observational data input (Appendix C), and the results are shown in Appendices D and E. Appendices D and E show that the

least-squares computer program either with all towers or with a tower omitted give results in deflection, range, and height of burst that are similar to the results of the triangulation program for rounds 11 through 17. Round 18, however, is an interesting case. Depending on the computer program, the range values for this round differ absolutely by some 20 meters. Which program does one believe in? The author believes the results of the least-squares computer program because it uses all the information and there is no statistical reason for eliminating any of the four tower azimuth readings.

Note that the least-squares computer program can incorporate time of flight observational measurements in the same manner as the triangulation program (Roberts 1990). Also, the least-squares computer program has been written for the Aberdeen Proving Ground (APG) where azimuths are measured clockwise from due south, and the Cartesian coordinate system at the gun has its x-axis oriented 35° clockwise from due south, with positive z-axis going into the ground. This program can easily be adapted to the coordinate systems of other proving grounds. The least-squares computer program is on the VAX 8600 and VAX 780 computers in the Launch and Flight Division of the U.S. Army Ballistic Research Laboratory, APG, MD.

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5. REFERENCES

Dixon, W. J., and F. J. Massey. Introduction to Statistical Analysis. New York: McGraw-Hill Book Company, Inc., New York, 1957.

Hoel, P. G., S. C. Port, and C. T. Stone. Introduction to Statistical Theory. New York: Houghton Mifflin Company, 1971.

Roberts, N. P. "Measurements of Range, Deflection, Time of Flight, and Height of Burst for Field Artillery Shell Method I Triangulation." BRL-MR-3854, U.S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, MD, September 1990.

Spiegel, M. Probability and Statistics. New York: McGraw-Hill Book Company, Inc., 1975.

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APPENDIX A:
LISTING OF INPUT TO LEAST-SQUARES COMPUTER PROGRAM;
DEFINITIONS OF INPUT TO LEAST-SQUARES COMPUTER PROGRAM;
DESCRIPTION OF OUTPUT TO LEAST-SQUARES COMPUTER PROGRAM;

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DEFINITIONS OF INPUT TO LEAST-SQUARES COMPUTER PROGRAM

XT,YT,ZT	Position coordinates of T-th tower (T .LE. 6) in APG grid system
ITOWER	Tower number (1,2,3,4,5,6)
IP	Any character other than a blank
X(7),Y(7),Z(7)	Position coordinates of weapon, APG system
AZDG,AZMG	Line of fire measured clockwise from south, in degrees and minutes
AZD,AZM	Tower azimuth readings measured from south, in degrees and minutes
EZD,EZM	Tower elevation readings of burst height,in degrees and minutes
ID1,ID2	Round identification

LISTING OF INPUT TO LEAST-SQUARES COMPUTER PROGRAM

All input is punched into a file using the following format.

Items	Format
XT,YT,ZT,ITOWER,IP	3F10.2,47X,I1,1X,A1
.	
.	
XN,YN,ZN,ITOWER,IP N .LE. 6	
Blank	
X(7),Y(7),Z(7),AZDG,AZMG	3F10.2,2F10.1
For N of Towers:	
AZD(I,J),AZM,EZD,EZM,ID1,ID2	2F7.2,2F7.2,2x,2A3
.	
If blank data use -999. for both AZD and EZD	
.	
.	
AZD(K,J)	
+999.	

DESCRIPTION OF OUTPUT TO LEAST-SQUARES COMPUTER PROGRAM

Rd. no.	Deflections mils	Deflections meters	Range meters	Hob meters	Erms-Az mils	Erms-El mils	Towers
2A3	F6.1	F6.1	F8.1	F7.1	F8.3	F8.3	ALL
2A3	F6.1	F6.1	F8.1	F7.1	F8.3*	F8.3	OMITING TOWER J
2A3	F6.1	F6.1	F8.1	F7.1	F8.3	F8.3*	OMITTING TOWER J'

* Line printed only if value is found significant in outlier subroutine

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APPENDIX B:
LISTING OF LEAST-SQUARES COMPUTER PROGRAM
WRITTEN FOR ABERDEEN PROVING GROUND, MD, BY NEAL ROBERTS

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C     PROGRAM TO FIND RANGE,DEFLECTION,AND HEIGHT OF BURST
C     OF FIRED ARTILLERY PROJECTILES BY USING A LEAST
C     SQUARES TECHNIQUE WHICH MINIMIZES ANGULAR ERROR.
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION AZ(201,7),ALX(201,7),AZD(201,7),AZM(201,7)
      DIMENSION EZD(201,7),EZM(201,7),X(7),Y(7),Z(7)
      DIMENSION AZMLS(201,7),EZMLS(201,7),ISB(7),CNT(3),PT(3)
      DIMENSION RNG(7,201),XCNT(7),YCNT(7),ZCNT(7)
      DIMENSION ERROR(7),ERRORE(7)
      DIMENSION ID1(201),ID2(201)
      WRITE(6,700)
      N=0
    1 READ(5,501) XT,YT,ZT,ITOWER,IP
      IF(IP.EQ.1H ) GO TO 2
      X(ITOWER)=XT
      Y(ITOWER)=YT
      Z(ITOWER)=ZT
      N=N+1
      GO TO 1
    2 READ(5,502) X(7),Y(7),Z(7),AZDG,AZMG
      AZ(1,7)=((AZDG+AZMG/60.D0)-35.D0)*.017453293
      IRDG=0
      DO 70 J=1,N
      K=1
      DO 70 I=1,200
      GO TO (30,60) K
   30 READ(5,1030) AZD(I,J),AZM(I,J),EZD(I,J),EZM(I,J),ID1(I),ID2(I)
   32 IF(AZD(I,J).EQ.-999.D0) GO TO 60
   33 IF(AZD(I,J).NE.+999.D0) GO TO 50
   34 IF(IRGD-I) 35,40,40
   35 IRDG=I
   40 K=2
      GO TO 60
   50 IF(EZD(I,J).EQ.-999.D0) GO TO 60
      AZ(I,J)=((AZD(I,J)+AZM(I,J)/60.D0)-35.D0)*.017453293
      ALX(I,J)=(EZD(I,J)+EZM(I,J)/60.D0)*.017453293
      GO TO 70
   60 AZ(I,J)=99999.D0
      ALX(I,J)=-999.D0
   70 CONTINUE
      IRDG=IRDG-1
   80 DO 81 I=1,IRDG
   81 AZ(I,7)=AZ(1,7)
C     WRITE HEADINGS
      WRITE(6,701)
      WRITE(6,702)
      DO 280 I=1,IRDG
      NHR=1
      J=1
      DO 90 K=1,N
      IF(AZ(I,K).EQ.99999.D0) GO TO 90
      NHR=NHR+1
      ISB(J)=K
      J=J+1
   90 CONTINUE
```

```
      GO TO (274,110,120,130,130,130,130),NHR
  274 WRITE(6,1000) ID1(I),ID2(I)
      GO TO 280
  110 ISB(2)=7
  120 CALL POINT(AZ,I,ISB,X,Y,PT)
      CNT(1)=PT(1)
      CNT(2)=PT(2)
      CALL RANGE(X(7),Y(7),AZ(1,7),CNT,RN,DEFL,DEFLM)
      CNT(1)=PT(1)
      CNT(2)=PT(2)
      IF(ISB(2).NE.7) GO TO 121
      KK=ISB(1)
      CALL RANGE(X(KK),Y(KK),AZ(1,7),CNT,RNG(KK,I),DEFL,DEFLM)
      DZ=RNG(KK,I)*TAN(ALX(I,KK))
      ZN=Z(KK)+DZ
      GO TO 122
  121 KK1=ISB(1)
      KK2=ISB(2)
      CALL RANGE(X(KK1),Y(KK1),AZ(1,7),CNT,RNG(KK1,I),DEFL,DEFLM)
      CALL RANGE(X(KK2),Y(KK2),AZ(1,7),CNT,RNG(KK2,I),DEFL,DEFLM)
      DZ1=RNG(KK1,I)*TAN(ALX(I,KK1))
      DZ2=RNG(KK2,I)*TAN(ALX(I,KK2))
      ZN1=Z(KK1)+DZ1
      ZN2=Z(KK2)+DZ2
      ZN=(ZN1+ZN2)/2.
  122 WRITE(6,1002) ID1(I),ID2(I),DEFLM,DEFL,RN,ZN
      GO TO 280
  130 NT=NHR-1
      CALL POINT(AZ,I,ISB,X,Y,PT)
      ISB(NHR)=ISB(NT)+1
      CNT(2)=PT(2)
      CNT(1)=PT(1)
      KK=ISB(1)
      CALL RANGE(X(KK),Y(KK),AZ(1,7),CNT,RNG(KK,I),DEFL,DEFLM)
      DZ=RNG(KK,I)*TAN(ALX(I,KK))
      PT(3)=-(Z(KK)+DZ)
      DO 140 IT=1,NHR
      CNT(3)=PT(3)
      CNT(2)=PT(2)
      CNT(1)=PT(1)
      ITER=ISB(IT)
      CALL LSTQR(CNT,ERMS,ITER,I,ISB,AZ,NT,X,Y,Z,ERMSEL,ALX,IQ,ID1,ID2)
      IF(IQ.EQ.1) GO TO 280
      XCNT(ITER)=CNT(1)
      YCNT(ITER)=CNT(2)
      ZCNT(ITER)=CNT(3)
      ERROR(ITER)=ERMS
      ERRORE(ITER)=ERMSEL
  140 CONTINUE
```

```
      JALL=ISB(NHR)
      ERMS=ERROR(JALL)
      CNT(1)=XCNT(JALL)
      CNT(2)=YCNT(JALL)
      CALL RANGE(X(7),Y(7),AZ(1,7),CNT,RN,DEFL,DEFLM)
      ERMSE=ERRORE(JALL)
      HOB=-ZCNT(JALL)
      WRITE(6,1003) ID1(I),ID2(I),DEFLM,DEFL,RN,HOB,ERMS,ERMSE
      IF(ERROR(JALL).EQ.0..AND.ERRORE(JALL).EQ.0.) GO TO 280
      IF(ERROR(JALL).EQ.0) GO TO 150
      CALL MODF(ERROR,ISB,NHR,NT,ITOWER,VAR1,VAR,III)
      IF(III.EQ.0) GO TO 150
      CALL OUTLY(VAR1,VAR,SIG,ITEST,NHR)
      IF(ITEST.EQ.0) GO TO 150
      J=ITOWER
      CNT(1)=XCNT(J)
      CNT(2)=YCNT(J)
      ERMS=ERROR(J)
      ERMSE=ERRORE(J)
      HOB=-ZCNT(J)
      CALL RANGE(X(7),Y(7),AZ(1,7),CNT,RN,DEFL,DEFLM)
      WRITE(6,1004) ID1(I),ID2(I),DEFLM,DEFL,RN,HOB,ERMS,ERMSE,J,SIG
  150 IF(ERRORE(JALL).EQ.0) GO TO 280
      CALL MODF(ERRORE,ISB,NHR,NT,ITOWER,VAR1,VAR,III)
      IF(III.EQ.0) GO TO 280
      CALL OUTLY(VAR1,VAR,SIG,ITEST,NHR)
      IF(ITEST.EQ.0) GO TO 280
      J=ITOWER
      CNT(1)=XCNT(J)
      CNT(2)=YCNT(J)
      ERMS=ERROR(J)
      ERMSE=ERRORE(J)
      HOB=-ZCNT(J)
      CALL RANGE(X(7),Y(7),AZ(1,7),CNT,RN,DEFL,DEFLM)
      WRITE(6,1005) ID1(I),ID2(I),DEFLM,DEFL,RN,HOB,ERMS,ERMSE,J,SIG
  280 CONTINUE
      STOP
  501 FORMAT(3F10.2,47X,I1,1X,A1)
  502 FORMAT(3F10.2,2F10.1)
  700 FORMAT('1')
  701 FORMAT('0','RD.NO.',T12,'DEFLECTIONS',T30,'RANGE',T40,'HOB',T55,'E
     1RMS-AZ',T65,'ERMS-EL',T80,'TOWERS')
  702 FORMAT(1H ,T11,'MILS',T19,'METERS',T39,'METERS',T57,'MILS',T67,'MI
     1LS')
 1000 FORMAT('0',A3,A3)
 1002 FORMAT('0',2A3,T9,F6.1,T19,F6.1,T28,F8.1,T38,F7.1)
 1003 FORMAT('0',2A3,T9,F6.1,T19,F6.1,T28,F8.1,T38,F7.1,T54,F8.3,T64,F8.
     13,T82,'ALL')
 1004 FORMAT(1H ,2A3,T9,F6.1,T19,F6.1,T28,F8.1,T38,F7.1,T54,F8.3,T62,'*'
     1,T64,F8.3,T80,'OMITING TOWER ',I1,' (SIG. AT ',F4.2,' LEVEL)')
 1005 FORMAT(1H ,2A3,T9,F6.1,T19,F6.1,T28,F8.1,T38,F7.1,T54,F8.3,T64,F8.
     13,T72,'*',T80,'OMITING TOWER ',I1,' (SIG. AT ',F4.2,' LEVEL)')
 1030 FORMAT(2F7.2,2F7.2,2X,2A3)
      END
```

```
SUBROUTINE POINT(AZ,I,ISB,X,Y,PT)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION AZ(201,7),ISB(7),X(7),Y(7),PT(3)
JISC=ISB(1)
KISC=ISB(2)
A=TAN(AZ(I,JISC))
B=TAN(AZ(I,KISC))
PT(1)=(X(JISC)*A-Y(JISC)+Y(KISC)-X(KISC)*B)/(A-B)
PT(2)=(PT(1)-X(KISC))*B+Y(KISC)
RETURN
END
```

```
      SUBROUTINE RANGE(X,Y,AZ,CNT,RNG,DEFL,DEFLM)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION CNT(3)
      RNG=CNT(1)-X
      DEFL=CNT(2)-Y
      CALL ATNSR(DEFL,RNG,BETA,1)
      DEFLM=BETA-AZ
   10 IF(DEFLM+3.14159265) 30,30,20
   20 IF(DEFLM-3.14159265) 50,40,40
   30 DEFLM=DEFLM+6.28318531
      GO TO 10
   40 DEFLM=DEFLM-6.28318531
      GO TO 10
   50 RNG=RNG/COS(BETA)
      DEFL=RNG*SIN(DEFLM)
      DEFLM=1018.59164*DEFLM
      RETURN
      END
```

```
      SUBROUTINE ATNSR(A,B,C,L)
      IMPLICIT REAL*8(A-H,O-Z)
      P=3.14159265
      C=ATAN(A/B)
      IF(A) 10,40,40
   10 IF(B) 20,30,30
   20 GO TO (21,22),L
   21 C=P+C
      GO TO 50
   22 C=C-2.*P
      GO TO 50
   30 GO TO (31,50),L
   31 C=2.*P+C
      GO TO 50
   40 IF(B) 20,50,50
   50 RETURN
      END
```

```
      SUBROUTINE LSTQR(CNT,ERMS,ITER,I,ISB,AZ,NT,X,Y,Z,ERMSEL,ALX,IQ,ID1
     1,ID2)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION AZ(201,7),ALX(201,7),CNT(3),ISB(7),X(7),Y(7),Z(7)
      DIMENSION DAZ(6),DAZX(6),DAZY(6),RNG(7,201)
      DIMENSION DEL(6),DELX(6),DELY(6),DELZ(6),ID1(201),ID2(201)
      ICOUNT=1
      IQ=0
  183 ICOUNT=ICOUNT+1
      IF(ICOUNT.NE.201) GO TO 10
      WRITE(6,1) ID1(I),ID2(I)
    1 FORMAT(1X,2A3,5X,'LSTQR SUBROUTINE DID NOT CONVERGE IN 200 ITERATI
     1ONS')
      IQ=1
      GO TO 9
   10 DO 190 II=1,NT
      J=ISB(II)
      IF(J.EQ.ITER) GO TO 190
      YC=CNT(2)-Y(J)
      XC=CNT(1)-X(J)
      CALL ATNSR(YC,XC,ANGLE,1)
      DAZ(II)=ANGLE-AZ(I,J)
  184 IF(DAZ(II)+3.14159265) 186,186,185
  185 IF(DAZ(II)-3.14159265) 189,188,188
  186 DAZ(II)=DAZ(II)+6.28318531
      GO TO 184
  188 DAZ(II)=DAZ(II)-6.28318531
      GO TO 184
  189 CONTINUE
      DAZX(II)=-YC/(XC**2+YC**2)
      DAZY(II)=XC/(XC**2+YC**2)
  190 CONTINUE
      N=0
      SERMS=0.
      DO 191 II=1,NT
      J=ISB(II)
      IF(J.EQ.ITER) GO TO 191
      N=N+1
      SERMS=SERMS+DAZ(II)**2
  191 CONTINUE
      ERMS=1018.59164*SQRT(SERMS/FLOAT(N))
      TM1=0.
      TM2=0.
      TM3=0.
      TM4=0.
      TM5=0.
      TM6=0.
      DO 192 II=1,NT
      J=ISB(II)
      IF(J.EQ.ITER) GO TO 192
      TM1=TM1+DAZ(II)*DAZX(II)
      TM2=TM2+DAZX(II)**2
      TM3=TM3+DAZX(II)*DAZY(II)
      TM4=TM4+DAZ(II)*DAZY(II)
      TM5=TM5+DAZX(II)*DAZY(II)
      TM6=TM6+DAZY(II)**2
  192 CONTINUE
```

```
      DY=(TM1*TM5-TM4*TM2)/(TM3*TM5-TM6*TM2)
      DX=(TM1-TM3*DY)/TM2
      DO 200 II=1,NT
      J=ISB(II)
      IF(J.EQ.ITER) GO TO 200
      YC=CNT(2)-Y(J)
      XC=CNT(1)-X(J)
      ZC=CNT(3)+Z(J)
      CALL RANGE(X(J),Y(J),AZ(1,7),CNT,RNG(J,I),DEFL,DEFLM)
      ANGLE=ATAN(ZC/RNG(J,I))
      DEL(II)=-ANGLE-ALX(I,J)
  194 IF(DEL(II)+3.14159265) 196,196,195
  195 IF(DEL(II)-3.14159265) 199,198,198
  196 DEL(II)=DEL(II)+6.28318531
      GO TO 194
  198 DEL(II)=DEL(II)-6.28318531
      GO TO 194
  199 CONTINUE
      D1=(XC**2+YC**2+ZC**2)
      D2=SQRT(XC**2+YC**2)
      DELX(II)=(-XC*ZC)/(D1*D2)
      DELY(II)=(-YC*ZC)/(D1*D2)
      DELZ(II)=D2/D1
  200 CONTINUE
      N=0
      SERMS=0.
      DO 201 II=1,NT
      J=ISB(II)
      IF(J.EQ.ITER) GO TO 201
      N=N+1
      SERMS=SERMS+DEL(II)**2
  201 CONTINUE
      ERMSEL=1018.59164*SQRT(SERMS/FLOAT(N))
      TM1=0.
      TM2=0.
      TM3=0.
      TM4=0.
      DO 202 II=1,NT
      J=ISB(II)
      IF(J.EQ.ITER) GO TO 202
      TM1=TM1+DEL(II)*DELZ(II)
      TM2=TM2+DELX(II)*DELZ(II)
      TM3=TM3+DELY(II)*DELZ(II)
      TM4=TM4+DELZ(II)**2
  202 CONTINUE
      DZ=(TM1-TM2*DX-TM3*DY)/TM4
      IF(ABS(DX).LE..1.AND.ABS(DY).LE..1.AND.ABS(DZ).LE..1) GO TO 9
      CNT(2)=CNT(2)-DY
      CNT(1)=CNT(1)-DX
      CNT(3)=CNT(3)+DZ
      GO TO 183
    9 RETURN
      END
```

```
      SUBROUTINE OUTLY(ERMS,ERROR,SIG,ITEST,NT)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION F99(8),F975(8),F95(8),F90(8)
      DATA F99/.0,.0,.0001,.0100,.0442,.0928,.1447,.1948/
      DATA F975/.0,.0,.0007,.0248,.0808,.1453,.2066,.2616/
      DATA F95/.0,.0,.0027,.0494,.1270,.2032,.2696,.3261/
      DATA F90/.0,.0,.0109,.0975,.1984,.2826,.3503,.4050/
      F=ERMS/ERROR
      IF(F.LE.F99(NT)) GO TO 10
      IF(F.LE.F975(NT)) GO TO 20
      IF(F.LE.F95(NT)) GO TO 30
      IF(F.LE.F90(NT)) GO TO 40
      ITEST=0
      RETURN
   10 SIG=.99
      ITEST=1
      RETURN
   20 SIG=.975
      ITEST=1
      RETURN
   30 SIG=.95
      ITEST=1
      RETURN
   40 SIG=.90
      ITEST=1
      RETURN
      END
```

```
      SUBROUTINE MODF(ERROR,ISB,NHR,NT,ITOWER,VAR1,VAR,III)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION ERROR(7),ISB(7)
      TEMP=ERROR(NHR)
      X1=ERROR(NHR)
      DO 160 IT=1,NT
      J=ISB(IT)
      ERMS=ERROR(J)
      IF(ERMS.GE.TEMP) GO TO 160
      ITOWER=J
      X1=ERROR(J)
      TEMP=ERROR(J)
  160 CONTINUE
      IF(X1.GE.ERROR(NHR)) GO TO 169
      SUM=0.D0
      DO 162 IT=1,NHR
      J=ISB(IT)
      SUM=SUM+ERROR(J)
  162 CONTINUE
      AVG=SUM/FLOAT(NHR)
      SUM=0.D0
      DO 164 IT=1,NHR
      J=ISB(IT)
      IF(J.EQ.ITOWER) GO TO 164
      SUM=SUM+ERROR(J)
  164 CONTINUE
      AVG1=SUM/FLOAT(NT)
      VAR=0.D0
      DO 166 IT=1,NHR
      J=ISB(IT)
      VAR=VAR+(ERROR(J)-AVG)**2
  166 CONTINUE
      VAR1=0.D0
      DO 168 IT=1,NHR
      J=ISB(IT)
      IF(J.EQ.ITOWER) GO TO 168
      VAR1=VAR1+(ERROR(J)-AVG1)**2
  168 CONTINUE
      III=1
      GO TO 170
  169 III=0
  170 RETURN
      END
```

APPENDIX C:
SAMPLE INPUT TO LEAST-SQUARES COMPUTER PROGRAM

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```
 3657.607 12801.626    32.13                                      1 T
 3868.530 13870.487    13.3                                       2 T
 3754.335 13094.383     4.77                                      3 T
 2883.79  11998.31     12.13                                      4 T

 3812.273 12475.803            41.0       26.
34.      22.      18.       51.       11
34.      23.      17.       45.       12
34.      25.      18.       46.       13
34.      18.      17.       41.       14
34.      11.      5.        35.       15
34.      17.      5.        09.       16
34.      05.      5.        35.       17
34.      17.      5.        08.       18
+999.
11.      08.      19.       07.       11
9.       54.      18.       01.       12
11.      13.      19.       05.       13
9.       42.      17.       42.       14
10.      52.      6.        09.       15
11.      19.      6.        37.       16
10.      36.      6.        01.       17
11.      03.      5.        32.       18
+999.
26.      57.      20.       04.       11
27.      28.      18.       50.       12
27.      52.      19.       45.       13
27.      24.      18.       40.       14
27.      42.      6.        45.       15
27.      57.      5.        55.       16
27.      32.      6.        35.       17
27.      45.      5.        50.       18
+999.
47.      37.      14.       48.       11
47.      55.      13.       38.       12
47.      35.      14.       56.       13
48.      03.      13.       41.       14
47.      25.      4.        35.       15
47.      24.      4.        19.       16
47.      33.      4.        39.       17
47.      35.      4.        17.       18
+999.
```

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APPENDIX D:
SAMPLE OUTPUT FROM LEAST-SQUARES COMPUTER PROGRAM

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RD.NO.	DEFLECTIONS MILS	DEFLECTIONS METERS	RANGE	HOB METERS	ERMS-AZ MILS	ERMS-EL MILS	TOWERS
11	-0.8	-2.0	2548.5	953.3	7.283	1.328	ALL
11	4.4	10.9	2553.7	951.7	0.274*	0.428	OMITING TOWER 3 (SIG. AT 0.95 LEVEL)
11	4.4	10.9	2553.7	951.7	0.274	0.428*	OMITING TOWER 3 (SIG. AT 0.98 LEVEL)
12	8.9	21.3	2428.3	853.2	1.266	1.702	ALL
12	10.8	25.6	2417.4	851.5	0.802	0.856*	OMITING TOWER 4 (SIG. AT 0.90 LEVEL)
13	3.8	9.6	2561.6	951.7	0.843	1.674	ALL
13	4.4	11.1	2562.2	954.0	0.074*	1.439	OMITING TOWER 3 (SIG. AT 0.95 LEVEL)
14	9.9	23.5	2405.8	839.5	0.472	0.885	ALL
14	9.4	22.2	2407.4	838.8	0.061*	0.570	OMITING TOWER 1 (SIG. AT 0.95 LEVEL)
15	0.5	1.2	2547.1	303.4	0.292	2.796	ALL
16	0.9	2.4	2587.4	293.7	0.273	8.210	ALL
16	0.8	1.9	2587.2	299.6	0.016*	8.713	OMITING TOWER 3 (SIG. AT 0.95 LEVEL)
16	0.8	2.0	2591.9	279.9	0.208	0.613*	OMITING TOWER 2 (SIG. AT 0.99 LEVEL)
17	0.8	2.0	2518.1	297.2	0.433	1.488	ALL
18	2.6	6.5	2550.1	274.3	0.853	0.791	ALL

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APPENDIX E:
SUMMARIZED DATA OUPUT TO TRIANGULATION COMPUTER PROGRAM USING SAMPLE DATA INPUT OF APPENDIX C

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RD.NO.	DEFLECTIONS MILS	DEFLECTIONS METERS	RANGE METERS	AVERAGE HOB METERS
11	4.3	10.9	2551.5	952.9
12	7.7	18.3	2433.0	853.9
13	4.4	11.1	2562.7	952.9
14	9.4	22.3	2407.5	840.2
15	1.0	2.6	2543.3	302.1
16	0.8	1.9	2587.1	292.8
17	0.2	0.5	2520.9	297.5
18	3.5	8.6	2530.4	273.0

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